A scheme to implement the Deutsch-Josza algorithm on a superconducting charge-qubit quantum computer*

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Received August 16, 2005; Revised October 20, 2005

Abstract We have studied the implementation of the Deutsch-Josza quantum algorithm in a superconducting charge-qubit quantum computer. Different from previous studies, we have used the inductance coupled system of You et al. The detailed pulse sequences have been designed for the four possible functions in a 2-qubit system. The result is generalized to an arbitrary n-qubit system. This scheme will be useful for practical implementation of the algorithm.

Keywords: superconducting, quantum computer, Deutsch-Josza algorithm.

In recent years, quantum information and quantum computation have developed rapidly[1,2]. Quantum computer is the computer which works according to the principle of quantum mechanics. Shor algorithm[3] and Grover algorithm[4] have shown the great potential of quantum computer in factoring large numbers and searching in an unsorted database. The Deutsch-Jozsa algorithm(D-J algorithm)[5] is a benchmark algorithm that demonstrates the power of quantum computation. D-J algorithm has been experimentally implemented in a 2-qubit nuclear magnetic resonance quantum computer[6,7], in 5-qubit and 7-qubit NMR quantum computer[8,9], and in an ion trap quantum computer[10].

Recently, much attention has been paid to implementing quantum computer utilizing solid-state devices such as quantum dots[11,12] and superconducting Josephson junctions[13]. Solid-state quantum computer has a superiority in scalability. The quantum computer based on superconducting Josephson effect has developed rapidly both in theoretical and experimental studies[13—18]. It is interesting to implement quantum algorithms in such a quantum computing scheme. An implementing scheme for the D-J algorithm on superconducting quantum computer has already been proposed by Siewert et al.[19] to implement the modified D-J algorithm[20]. In their proposal, they adopted the capacitance coupling scheme in which the inter-qubit coupling has the form $\sigma_z\sigma_z$[21] to implement 2-qubit gates. In addition to coupling two qubits using capacitance, one can couple the two qubits using inductance[16]. In this work, we will study the implementation of D-J algorithm in the inductance coupled two charge-qubit system. The result will be generalized into systems with arbitrary n qubits.

1 The flux and voltage controlled superconducting quantum computer with Josephson charge qubits

A simple Josephson charge qubit is depicted in Fig. 1(a). It consists of a small superconducting box with n excess Cooper-pairs, connected to a superconducting electrode by a tunnel junction with capacitance C_J and coupling energy E_J. The superconducting box is biased by a gate voltage through a gate capacitor C_g. There are two energy scales, the Cooper-pair charging energy $E_c=(2e)^2/2(C_g+C_J)$, and the Josephson coupling energy E_J, which is proportional to the critical current of the Josephson Junction.

Choosing suitable materials and parameters satisfying that the superconducting energy gap $\Delta > E_c$ and $E_c \gg E_J$, then at low temperature, $k_B T \ll E_J$ (where k_B is the Boltzmann constant), there are only two charge states, $n=0$ and $n=1$, playing a role. All the other states having much higher energy are thus ignored. In this case, the Hamiltonian of the system

* Supported by the National Fundamental Research Program (Grant No. 001CB309308), National Natural Science Foundation of China (Grant Nos. 10325521, 60433050), the Hang-Tian Science Fund, and the SRFDP program of Education Ministry of China.

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can be written as a spin-$\frac{1}{2}$ system

$$H = -\frac{1}{2}B_z\sigma_z - \frac{1}{2}B_x\sigma_x, \tag{1}$$

where

$$B_z = E_c(1 - 2n_g),$$
$$B_x = E_J,$$
$$n_g = \frac{C_g V_g}{2e}. \tag{2}$$

Here, n_g is the offset charge which can be controlled by gate voltage. The charge states $n = 0$ and $n = 1$ correspond to the spin basis $|0\rangle = |\uparrow\rangle = \begin{pmatrix}1\\0\end{pmatrix}$ and $|1\rangle = |\downarrow\rangle = \begin{pmatrix}0\\1\end{pmatrix}$, respectively.

The simple single Josephson junction is difficult to operate because the tunnelling amplitude, the x component of the field B_x in Hamiltonian Eq. (1), is constant. Thus, in manipulating the system, it is not only necessary to control the operating time, but also to keep track of the time t_0 from the very beginning of manipulating. To solve this difficulty, people have replaced the single Josephson junction by two junctions placed in a loop configuration as shown in Fig. 1 (b)[15]. This forms a dc SQUID. This dc SQUID is biased by an external magnetic flux Φ_x, and the tunnelling amplitude, or the effective Josephson coupling energy, is controllable by changing the external flux. The explicit expression is

$$B_x = 2E_J\cos\left(\pi\frac{\Phi_x}{\Phi_0}\right), \tag{3}$$

where $\Phi_0 = \frac{hc}{2e}$ is the flux quantum. Consequently, the SQUID-controlled qubit is described by the following Hamiltonian

$$H = -\frac{1}{2}B_z(V_g)\sigma_z - \frac{1}{2}B_x(\Phi_x)\sigma_x, \tag{4}$$

with field components $B_z = E_c(1 - 2n_g)$ and $B_x(\Phi_x) = 2E_J\cos\pi\frac{\Phi_x}{\Phi_0}$ controlled independently by the gate voltage and the external flux.

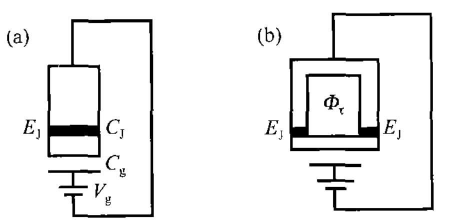

Fig. 1. The simplest Josephson charge qubit design formed by a superconducting single-charge box (a), and the Josephson charge qubit with controlled tunnelling amplitude (b).

A scalable charge-qubit quantum computer scheme is proposed by You et al. in Ref. [16]. There are also other ways to couple different charge qubits[14,15,19]. The scheme in Ref. [16] realizes the coupling of different qubits via a common superconducting inductance L. This scheme is more efficient in performing two-qubit conditionale gates, because it requires just one two-qubit operation to perform conditional gates. We will work in this superconducting quantum computer scheme. The n-qubit circuit is shown in Fig. 2. In this model, the superconducting box is coupled by two symmetric dc SQUIDs, and each SQUID is pierced by a magnetic flux Φ_{xi} where the subscript refers to the i-th qubit. The two charge levels in the superconducting box serve as the two states of a qubit. The Hamiltonian of one qubit reads[16]

$$H = -\frac{1}{2}B_z(V_g)\sigma_z - \frac{1}{2}B_x(\Phi_x, \Phi_e)\sigma_x. \tag{5}$$

Here,

$$B_z(V_g) = E_c(1 - 2n_g) = E_c\left(1 - \frac{C_g V_g}{e}\right),$$
$$B_x(\Phi_x, \Phi_e) = 4E_J^0\cos\left(\pi\frac{\Phi_x}{\Phi_0}\right)\cos\left(\pi\frac{\Phi_e}{\Phi_0}\right) \cdot \left(1 - \frac{1}{2}\eta^2\sin^2\left(\pi\frac{\Phi_e}{\Phi_0}\right)\right),$$

where $\eta = -\pi^2 L E_J(\Phi_x)/\Phi_0$. If $C_g V_g/e = 1$ and $\Phi_x/\Phi_0 = 1/2$, $H = 0$. This state may be called an immune state of the qubit because the qubit does not change its state. There is no time evolution for this qubit. If these conditions are not satisfied, the qubit will leave from the immune state and go through a time evolution. Thus, by changing the parameters Φ_x (the magnetic flux), and V_g (the gated voltage), one can address a specific qubit and make it go through a designated time evolution. This fulfils a single qubit addressing and operation for quantum computation.

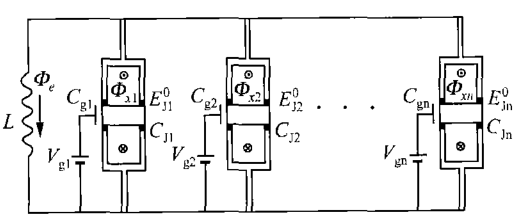

Fig. 2. Schematic diagram of n-qubit circuit. The superconducting box is coupled by two symmetric dc SQUIDs. These two dc SQUIDs are identical and all Josephon junctions in one qubit have coupling energy E_{Ji}^0 and capacitance C_{Ji}. Each SQUID in one qubit is pierced by a magnetic flux Φ_{xi} but the direction of the flux is opposite.

The two-qubit gate is easy to implement in this scheme[16]. The inter-qubit interaction is introduced by the common inductance L through the electromagnetic energy in the inductance $1/2LI^2$, where $I = \sum_{i=1}^{N} I_{ci}$ is the total electric current through the inductance. $I_{ci} = -\pi E_{Ji}(\Phi_{xi})/\Phi_0$ is the contribution to the current from the i-qubit. When $\Phi_{xi}/\Phi_0 = 1/2$, the contribution from the i-th qubit is zero. Hence, if all qubits are in their immune state condition, namely $\Phi_{xi}/\Phi_0 = 1/2$ and $C_g V_{gi}/e = 1$, there is no interaction among the qubits. To realize a two-qubit gate, one needs to depart from this immune state. By setting $\Phi_{xk} = \frac{\Phi_0}{2}$ and $V_{gk} = \frac{e}{C_{gk}}$ for all qubits except $k = i$ and j, the i-th qubit and j-th qubit can be coupled together through the inductance, and the Hamiltonian for the whole computer contains only contributions from these two qubits. After some derivation, it can be written as[16]

$$H = \sum_{k=i,j}\Big[-\frac{1}{2}B_z^{(k)}(V_{gk})\sigma_z^{(k)} - \frac{1}{2}B_x^{(k)}(\Phi_{xk}, \Phi_e)\sigma_x^{(k)}\Big] + E_{\text{int}}\sigma_x^{(i)}\sigma_x^{(j)}, \quad (6)$$

where

$$E_{\text{int}} = -L\frac{4\pi^2}{\Phi_0^2}E_{Ji}^{(0)}E_{Jj}^{(0)}\cos\left(\pi\frac{\Phi_{xi}}{\Phi_0}\right) \cdot \cos\left(\pi\frac{\Phi_{xj}}{\Phi_0}\right)\sin^2\left(\pi\frac{\Phi_e}{\Phi_0}\right). \quad (7)$$

Here, E_{Ji}^0 is the Josephson coupling energy of the ith qubit, Φ_{xi} is the magnetic flux threading the dc SQUID of the ith qubit, and Φ_e is the magnetic flux threading the superconducting inductance L.

The basic gate operations are one bit operation and the two-qubit controlled phase gate. A quantum system evolves according to $U(t) = e^{-iHt/\hbar}$. Initially, setting $\Phi_{xi} = \frac{1}{2}\Phi_0$ and $V_{gi} = \frac{2e}{C_{gi}}$ $(i = 1, 2)$ so that the Hamiltonian of the system is $H = 0$ and no time evolution occurs. It can implement logic gates by switching certain magnetic flux Φ_{xi} and/or gate voltage V_{gi} away from the initial values for certain periods of times. The universal set of one-bit gates $U_z(\alpha) = e^{i\alpha\sigma_z}$ and $U_x(\beta) = e^{i\beta\sigma_x}$, where $\alpha = \frac{B_z(V_g)\tau}{2\hbar}$ and $\beta = \frac{B_x(\Phi_x, \Phi_e)\tau}{2\hbar}$, can be designed by choosing $B_x(\Phi_x, \Phi_e) = 0$, $B_z(V_g) = B_z \neq 0$ and $B_z(V_g) = 0$, $B_x(\Phi_x, \Phi_e) = B_x \neq 0$ in the one bit Hamiltonian Eq. (5) for a given time τ, respectively. Any one-bit operation can be derived with these two one-bit gates. For example, the Hadamard transformation H and the one bit rotation $U_y\left(\frac{\pi}{4}\right) = e^{i\frac{\pi}{4}\sigma_y}$ are given by $H = e^{-i\frac{\pi}{2}}U_z\left(\frac{\pi}{4}\right)U_x\left(\frac{\pi}{4}\right)U_z\left(\frac{\pi}{4}\right)$ and $U_y\left(\frac{\pi}{4}\right) = e^{i\frac{\pi}{4}\sigma_y} = U_z\left(-\frac{\pi}{4}\right)U_x\left(\frac{\pi}{4}\right)U_z\left(\frac{\pi}{4}\right)$, respectively[16]. Here, the phase factor $e^{-i\frac{\pi}{2}}$ corresponds to a total energy shift of the Hamiltonian.

When the fluxes Φ_{xi} and Φ_{xj} are switched away from the initial value $\Phi_0/2$ for a given period time τ, the Hamiltonian of the two qubits becomes $H(t) = -\frac{1}{2}B_x^{(i)}\sigma_x^{(i)} - \frac{1}{2}B_x^{(j)}\sigma_x^{(j)} + E_{\text{int}}\sigma_x^{(i)}\sigma_x^{(j)}$. If the parameters are suitably chosen so that $\frac{1}{2}B_x^{(i)} = \frac{1}{2}B_x^{(j)} = E_{\text{int}} = -\frac{\pi\hbar}{4\tau}$, a controlled-phase-gate is reached, $U_p' = e^{i\frac{\pi}{4}}e^{-iH(t)\tau/\hbar} = e^{i\frac{\pi}{4}[1-\sigma_x^{(i)}-\sigma_x^{(j)}+\sigma_x^{(i)}\sigma_x^{(j)}]}$, which does not change the 2-qubit states $|+\rangle|+\rangle$, $|+\rangle|-\rangle$ and $|-\rangle|+\rangle$, but transforms $|-\rangle|-\rangle$ to $-|-\rangle|-\rangle$, where $|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$. The controlled-phase-shift gate for the basis states $|0\rangle|0\rangle$, $|0\rangle|1\rangle$, $|1\rangle|0\rangle$ and $|1\rangle|1\rangle$ can be obtained by combining U_p' with Hadamard transformation, $U_p = H^{(j)}H^{(i)}U_p'H^{(i)}H^{(j)}$. Then the controlled-not gate can be derived,

$$U_{\text{Cnot}} = U_y^{(j)}\left(-\frac{\pi}{4}\right)U_p U_y^{(j)}\left(\frac{\pi}{4}\right)^{[16]}.$$

2 Implementation of the D-J algorithm in superconducting charge-qubit quantum computer

The D-J algorithm determines whether a function $f(x)$ is constant or balanced[6]. Consider that there are n-bit inputs x, the function $f(x)$ is called constant if $f(x) = 0$ or 1 for all inputs x; and called balanced function if $f(x) = 0$ for exactly half the inputs and $f(x) = 1$ for the other half. To determine whether the function $f(x)$ is constant or balanced on a deterministic classical computer, in the worst case, $2^{n-1} + 1$ function calls are required: although half of the inputs have been checked and the value of function $f(x) = 0$ has been obtained, it cannot be concluded with certainty that the function $f(x)$ is constant and one additional call of the function is still necessary. In contrast, a quantum computer can certainly determine the property of $f(x)$ using just one function call using the D-J algorithm.

To realize an n qubit D-J algorithm in a quantum computer, one needs $n+1$ qubits. The extra qubit is used as an ancillary qubit to affect the tested function. Note that this tested function $f(x)$ is either a constant function or a balanced function. If the function is different from these two types of functions, then the D-J algorithm will fail. In a quantum computer, a function call is realized by a sequence of unitary gate operations, denoted by U_f. In the beginning, the state of $n+1$ qubits register is prepared in the following superposition state

$$\left(\frac{1}{2^{n/2}}\sum_{x=0}^{2^n-1}|x\rangle\right)\frac{1}{\sqrt{2}}(|0\rangle-|1\rangle).$$

The action of U_f is

$$U_f:\left(\frac{1}{2^{n/2}}\sum_{x=0}^{2^n-1}|x\rangle\right)\frac{1}{\sqrt{2}}(|0\rangle-|1\rangle)$$
$$\xrightarrow{U_f}\left(\frac{1}{2^{n/2}}\sum_{x=0}^{2^n-1}(-1)^{f(x)}|x\rangle\right)\frac{1}{\sqrt{2}}(|0\rangle-|1\rangle). \tag{8}$$

The state of the first n qubits,

$$\left(\frac{1}{2^{n/2}}\sum_{x=0}^{2^n-1}(-1)^{f(x)}|x\rangle\right)$$

will be changed to

$$|i_1 i_2\cdots i_n\rangle$$

after applying the Hadamard gate to the first n qubits. Through simple calculation, it can be concluded that if the function $f(x)$ is constant, then the state of the first n qubits would be $|00\cdots0\rangle$; however, if the function $f(x)$ is balanced, then the probability of being $|00\cdots0\rangle$ state would be 0. Therefore, whether the function is constant or balanced can be determined by measuring the state of the first n qubits. If the state of the first n qubits is $|00\cdots0\rangle$, then the function is constant, otherwise the function is balanced.

In order to implement the D-J algorithm, the most important step is to design the operator U_f. There are two constant functions for n-qubit D-J algorithm, one is $f(x)=0$ and the other is $f(x)=1$. It is easy to design operators U_f corresponding to these two functions. The operator U_f corresponding to $f(x)=0$ is the identity operator. The operator U_f corresponding to $f(x)=1$ is $U_x^n\left(\frac{\pi}{2}\right)$ (the superscript n denotes the n-th qubit). However, there are many balanced functions for a n-qubit system, and it is more difficult to design the operators U_f corresponding to these balanced functions. We will analyze the balanced functions in detail[16].

For 1-qubit D-J algorithm, there are two balanced functions, namely: $f_1(0)=0$, $f_1(1)=1$, and $f_2(0)=1$, $f_2(1)=0$. These two functions can be implemented by operators $U_{f1}=U_{\text{Cnot}}^{12}$ and $U_{f2}=U_{\text{Cnot}}^{12}U_x^2\left(\frac{\pi}{2}\right)$, where the numbers in the superscripts 1 and 2 denote the control qubit and the target qubit, respectively.

For a 2-qubit D-J algorithm, there are six balanced functions. For instance, $f_1(x)=0$ for $x=0$, 1 and $f_1(x)=1$ for $x=2,3$. f_1 is a balanced function, and it can be implemented through the following gate operations in the superconducting computer, $U_{f1}=U_{\text{Cnot}}^{13}$. Similarly, the other five balanced functions can be implemented by the following operators:

$$U_{f2}=U_{\text{Cnot}}^{23},\quad U_{f3}=U_{\text{Cnot}}^{13}U_x^3\left(\frac{\pi}{2}\right),$$
$$U_{f4}=U_{\text{Cnot}}^{23}U_x^3\left(\frac{\pi}{2}\right),\quad U_{f5}=U_{\text{Cnot}}^{13}U_{\text{Cnot}}^{23},$$
$$U_{f6}=U_{\text{Cnot}}^{23}U_{\text{Cnot}}^{13}U_x^3\left(\frac{\pi}{2}\right),$$

respectively. These operators can be constructed directly by managing the magnetic flux and the gating voltage described in the previous section.

For 3-qubit D-J algorithm, there are 70 balanced functions. These 70 functions can be implemented by combining the following basic operators, U_{Cnot}^{14}, U_{Cnot}^{24}, U_{Cnot}^{34}, $U_{\text{Cnot}}^{14}U_x^4\left(\frac{\pi}{2}\right)$, $U_{\text{Cnot}}^{24}U_x^4\left(\frac{\pi}{2}\right)$, and $U_{\text{Cnot}}^{34}U_x^4\left(\frac{\pi}{2}\right)$.

For 4-qubit D-J algorithm, there are 12870 balanced functions. Though the number of balanced functions is so big, they all can be implemented by combining the basics operators, U_{Cnot}^{15}, U_{Cnot}^{25}, U_{Cnot}^{35}, U_{Cnot}^{45}, $U_{\text{Cnot}}^{15}U_x^5\left(\frac{\pi}{2}\right)$, $U_{\text{Cnot}}^{25}U_x^5\left(\frac{\pi}{2}\right)$, $U_{\text{Cnot}}^{35}U_x^5\left(\frac{\pi}{2}\right)$, and $U_{\text{Cnot}}^{45}U_x^5\left(\frac{\pi}{2}\right)$.

It can be proved that there are $\binom{2^n}{2^{n-1}}$ balanced functions in an n-qubit system. When the number n increases, the number of balanced functions increases binomially in the exponential of n. However, each of the balanced function can be implemented by the basic operators, $U_{\text{Cnot}}^{1n+1}, \cdots, U_{\text{Cnot}}^{nn+1}, U_{\text{Cnot}}^{1n+1}U_x^{n+1}\left(\frac{\pi}{2}\right), \cdots$, and $U_{\text{Cnot}}^{nn+1}U_x^{n+1}\left(\frac{\pi}{2}\right)$.

There are five steps to perform in order to implement the n qubit D-J algorithm in a superconducting quantum computer:

① Prepare the initial state $|00\cdots 1\rangle$ where the ancillary qubit is originally prepared in 1.

② Apply the Hadamard gate on the $n+1$ qubits.

③ The tested function $f(x)$ is translated in a quantum circuit gate, which is an operator U_f on the $n+1$ qubits. These quantum circuit can be implemented by the basic gate operators in the superconducting quantum computer. In this charge-qubit superconducting quantum computer, the basic gates are one qubit gates $U_z(\alpha)$ and $U_x(\alpha)$, and the two-qubit controlled NOT gate,

$$U_{\text{Cnot}} = U_y^{(j)}\left(-\frac{\pi}{4}\right)U_p U_y^{(j)}\left(\frac{\pi}{4}\right).$$

④ Apply the Hadamard gate on the first n qubit.

⑤ Measure the states of the first n qubits. If the result is that all the qubits are in the 0 state, then $f(x)$ is a constant function. Otherwise it is a balanced function.

3 Error analysis

Experimentally, the charging energy E_c and Josephson coupling energy E_J cannot be measured precisely, there are always some errors ΔE_c and ΔE_J in E_c and E_J, respectively. The final state certainly will not be the correct state but with some probabilities because of the existence of the errors. In 2-qubit D-J algorithm, the operator U_{f3} is the most complicated one. Therefore, in order to see how these errors influence the final results of the D-J algorithm, we can calculate the probability of state $\frac{(|0\rangle-|1\rangle)(|0\rangle-|1\rangle)}{2}$ in the final state $|\Psi\rangle = U_{f3}H^{(1)}H^{(2)}U_x^{(2)}|00\rangle$.

For simplicity, we consider the four following cases in 1-qubit D-J algorithm: the first one is that there are errors in charging energy of the two qubits while the Josephson coupling energy is constant; the second one is that there are errors in Josephson coupling energy of the two qubits while the charging energy is constant; the third one is that there are errors in charging energy of the first qubit and in Josephson coupling energy of the second qubit; the last one is that there are errors in Josephson coupling energy of the first qubit and in charging energy of the second qubit. The probability distributions with δ_1 and δ_2 are shown in Figs. (3)—(6). δ_1 and δ_2 have different meanings in Figs. (3)—(6). The vertical coordinate P is the probability of success of D-J algorithm. In Fig. 3, $\delta_1 = \Delta E_{c1}/E_{c1}$ and $\delta_2 = \Delta E_{c2}/E_{c2}$; in Fig. 4, $\delta_1 = \Delta E_{J1}/E_{J1}$ and $\delta_2 = \Delta E_{J2}/E_{J2}$; in Fig. 5, $\delta_1 = \Delta E_{c1}/E_{c1}$ and $\delta_2 = \Delta E_{J2}/E_{J2}$; in Fig. 6, $\delta_1 = \Delta E_{J1}/E_{J1}$ and $\delta_2 = \Delta E_{c2}/E_{c2}$.

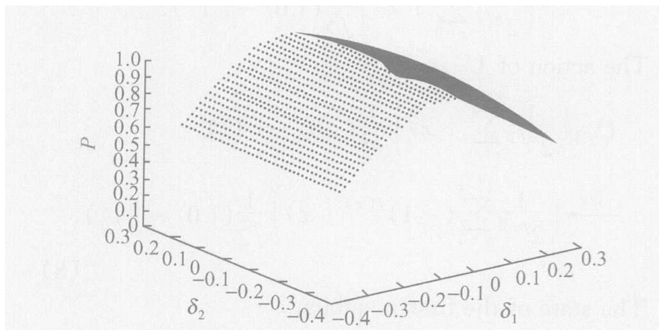

Fig. 3. The probability distribution with the errors in charging energy of two qubits.

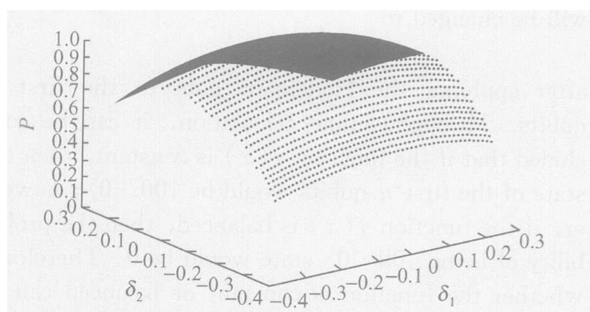

Fig. 4. The probability distribution with the errors in Josephson coupling energy of two qubits.

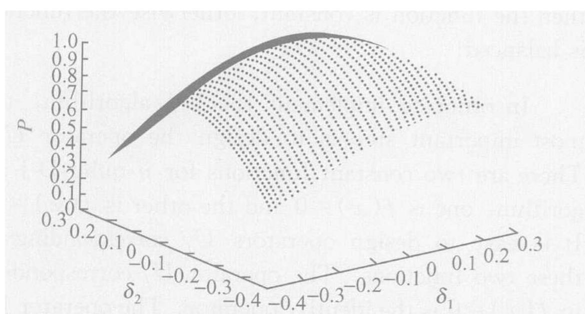

Fig. 5. The probability distribution with the errors in charging energy of the first qubit and Josephson coupling energy of the second qubit.

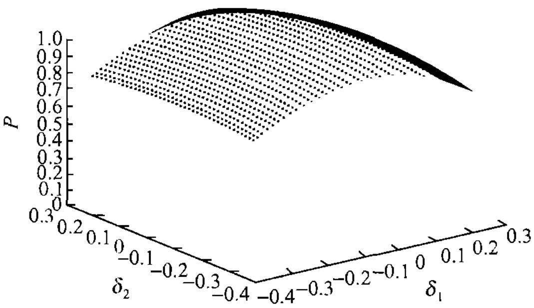

Fig. 6. The probability distribution with the errors in Josephson coupling energy of the first qubits and charging energy of the second qubit.

From these four figures we can see that the error in the second qubit, $\delta_2 = \Delta E_{J2}/E_{J2}$ influences the probability severely while $\delta_2 = \Delta E_{c2}/E_{c2}$ influences the probability slightly, and the situation is reversed in the first qubit. The influence of these errors is not symmetrical with the first qubit and the second qubit, since the operations on the first qubit and second qubit in operator U_{f3} are not equal. In Fig. 5, the probability decreases to a small value at larger $|\delta_1|$ and $|\delta_2|$, indicating that the result of D-J algorithm cannot be very good in the presence of larger ΔE_{c1} and ΔE_{J2}. Therefore, the error ΔE_{J2} should be very small in order to achieve good results of D-J algorithm.

4 Conclusion

We have studied the implementation of the D-J algorithm using flux-voltage-controlled superconducting charge qubit quantum computer. This result complements the case where the coupling between qubits is realized by capacitance. The detailed operating sequences have been designed. Results have shown that some of the functions involve very little inter-qubit gates, such as the two constant functions. Though these functions are also part of the D-J quantum algorithm, they are not good to test the practical performance of the quantum computer. Some balanced functions involve more complicated inter-qubit gate operations and are more suitable for testing quantum computer performance. By error analysis, we have found that the error of charging energy in controlled qubit and the error of Josephson coupling energy in ancillary qubit influence the final result more severely.

In our work, the qubits are coupled by a superconducting inductance L. There is another coupling mode called capacitance coupling. These two coupling mode have been experimentally realized[18,22]. Compared with inductance coupling, the capacitance coupling is easy to be realized when the number of qubits is very little. However, when the number of qubits is large, the additional operation and decoherence will be the biggest trouble of capacitance coupling model, and the advantage of inductance coupled superconducting quantum computer will become apparent.

References

1 Nielsen M. A. and Chuang I. L., Quantum Computation and Quantum Information, Cambridge: Cambridge University Press, 2000.

2 Bennett C. H. Quantum computation. Science, 1995, 270: 255—262.

3 Shor P. Algorithms for quantum computation: discrete algorithm and factoring. In: Proc. of the 35th Annual Sysmposium on Foundations of Computer Science, New York: IEEE Computer Science Press, 1994, 124—134.

4 Grover L. K. Quantum mechanics helps in searching for a needle in a haystack. Phys. Rev. Lett., 1997, 79(2): 325—328.

5 Deutsch D. and Jozsa R. Rapid solution of problems by quantum computation. Proc. R. Soc. London, Ser. A, 1992, 439: 553—558.

6 Jones J. A. and Mosca M. Implementation of a quantum algorithm on a nuclear magnetic resonance quantum computer. J. Chem. Phys., 1998, 109: 1648—1653.

7 Chuang I. L., Vandersypen L. M. K., Zhou X. et al. Experimental realization of a quantum algorithm. Nature, 1998, 393: 143—146.

8 Wei D., Luo J., Yang X. et al. NMR experimental realization of seventh-order coupling transformation and the seven-qubit modified Deutsch-Josza algorithm. http://xxx.lanl.gov/PS_cache/quant-ph/pdf/0301/0301041.pdf[2005-01-06].

9 Marx R., Fahmy A. F., Myers J.M. et al. Appraching five-bit NMR quantum computing. Phys. Rev. A, 2000, 62: 012310-1—8.

10 Gulde S., Riebe M., Lancaster G. P. T. et al. Implementation of the Deutsch-Jozsa algorithm on an ion-trap quantum computer. Nature, 2003, 421: 48—50.

11 Loss D. and DiVincenzo D. P. Quantum computation with quantum dots. Phys. Rev. A, 1998, 57: 120—126.

12 Kane B. E. A silicon-based nuclear spin quantum computer. Nature, 1998, 393: 133—137.

13 Makhlin Y., Schön G. and Shnirman A. Quantum-state engineering with Josephson-junction devices. Rev. Mod. Phys., 2001, 73: 357—400.

14 Shnirman A., Schön G. and Hermon Z. Quantum manipulations of small Josephson junctions. Phys. Rev. Lett., 1997, 79: 2371—2374.

15 Makhlin Y., Schön G. and Shnirman A. Josephson-junction qubits with controlled couplings. Nature, 1999, 398: 305—307.

16 You J. Q., Tsai J. S. and Nori F. Scalable quantum computing with Josephson charge qubits. Phys. Rev. Lett., 2002, 89: 197902-1—4.

17 Nakamura Y., Pashkin Y. A. and Tsai J. S. Coherent control of macroscopic quantum states in a single-Cooperpair box. Nature, 1999, 398: 786—788.

18 Yamamoto T., Pashkin Y. A., Astafiev O. et al. Demonstration of conditional gate operation using superconducting charge qubits. Nature, 2003, 425: 941—944.

19 Siewert J. and Fazio R. Quantum algorithms for quatnum networks. Phys. Rev. Lett., 2001, 87: 257905-1—4.

20 Collins D., Kim K. W. and Holton W. C. Deutsch-Jozsa algorithm as a test of quantum computation. Phys. Rev. A, 1998, 58: R1633—R1636.

21 Siewert J., Fazio R., Palma G. M. et al. Aspect of qubit dynamics in the presence of leakage. J. Low Temp. Phys., 2000, 118: 795—804.

22 Xu H. Z., Frederick W. S., Dutta S. K. et al. Spectroscopy of three-particle entanglement in a macroscopic superconducting circuit. Phys. Rev. Lett., 2005, 94: 027003-1—4.